Student No.: ________________________

Qualifying/Placement Exam, Part-B
2:00 – 4:00, January 3, 2019

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^9\ \mathrm{Nm^2/C^2}$	electric force constant
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{ev\cdot s}$	"
$c = 3.0\times10^8\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\ \mathrm{C}$	charge of the electron

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1. [10 pts] A particle of mass m moves in a one-dimensional box with the following potential

$$V = \begin{Bmatrix} \infty, & x < 0 \\ 0, & 0 \le x \le a \\ \infty, & x > a \end{Bmatrix}.$$

[5 pts] (a) Solve for the eigenfunction and energy eigenvalues of the particle.

[5 pts] (b) At $t = 0$, the wave function of the particle can be expressed as $\psi = \cos^2\left(\frac{\pi x}{a}\right)$ for $0 \le x \le a$. Calculate the possibility for the particle to be in the ground state.

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2. [10 pts] A particle of mass m moving in the positive x direction with momentum p, is incident on a potential step (down),

$$V = \begin{cases} 0, & x < 0 \\ -V_0, & x > 0 \end{cases}$$

where V_0 is a positive constant.

Draw this potential, and find the probability that the particle is reflected by the step.

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3. [10 pts] Two observables $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$ can be represented by matrices

$$A = a\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad B = b\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

[3 pts] a) If $\hat{\mathrm{A}}$ is measured, and the result $-a$ is observed, what is the state of the system (written as a two-component column vector of coefficients)?

[3 pts] b) Immediately after the measurement in part a), the observable $\hat{\mathrm{B}}$ is measured. What are the possible results that can be obtained from this measurement and with what probabilities? (Hint: obtain eigenvalues and normalized eigenvectors)

[4 pts] c) Now, immediately after the measurement in part b), the observable $\hat{\mathrm{A}}$ is measured again. What is the probability to obtain the result $-a$ again from this measurement? [Note that you are not given the outcome of the measurement in part b).]

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4. [10 pts] Determine the speed of the Lorentz transformation in the x-direction from the frame S, where the velocity of a particle is, $\mathbf{u} = (u_x, u_y) = \left(\frac{c}{\sqrt{2}}, \frac{c}{\sqrt{2}} \right)$, to the frame S' , where the velocity is $\mathbf{u}' = \left(\frac{-c}{\sqrt{2}}, \frac{c}{\sqrt{2}} \right)$.

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Billions of years ago the Solar System was created out of the remnants of a supernova explosion. Nuclear scientists believe that two isotopes of uranium, ${}^{235}_{92}\mathrm{U}$ and ${}^{238}_{92}\mathrm{U}$, were created in equal amounts at that time. However, today 99.28% of uranium is in the form of ${}^{238}_{92}\mathrm{U}$. The half-life of ${}^{235}_{92}\mathrm{U}$ and ${}^{238}_{92}\mathrm{U}$ are $2.22 \cdot 10^{16}$ s and $1.41 \cdot 10^{17}$ s, respectively

5. [10 pts] Assuming a simplified model in which all of the matter of the Solar System originated from a single exploding star, calculate the time (in the past) of this explosion.

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6. [10 pts] The differential cross section, $\frac{d\sigma}{d\Omega}$, that will cause particles to scatter at an angle of 55.0° off a target is $4.00 \cdot 10^{-18}$ m^2/sr. A detector with an area of 1.00 cm^2 is placed 1.00 m away from the target to detect particles that have been scattered at 55.0°. $3.00 \cdot 10^{17}$ particles hit the 1.00-mm^2-area target every second.

[5 pts] a) What is the solid angle subtended by the detector in steradians (sr)?

[5 pts] b) How many particles per second will strike the detector?