Student No.: _____

Qualifying/Placement Exam, Part-B
2:00 – 4:00, August 17, 2017, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{eV\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\ \mathrm{C}$	charge of the electron
$N_A = 6.022\times10^{23}\ \mathrm{mol^{-1}}$	Avogadro constant

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1. [10 pts] A particle of mass m moves in a two-dimensional infinite square well of dimension, a x a. The potential is given as:

$$V(x,y) = \begin{cases} 0 & \text{for } (0 < x < a) \,\&\, (0 < y < a) \\ \infty & \text{everywhere else} \end{cases}$$

a) [3 pts] What are the energy eigenfunctions, $\psi(x,y)$?

b) [3 pts] What are the associated energy eigenvalues?

c) [4 pts] List the three lowest eigenvalues, and the degeneracy of each.

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2. [10 pts] At time $t = 0$, a particle in a harmonic oscillator potential $V(x) = \frac{m\omega^2 x^2}{2}$ has a wavefunction $\psi(x) = \left(\frac{1}{\sqrt{2}}\right)\left(\psi_0(x) + \psi_1(x)\right)$, where $\psi_0(x)$ and $\psi_1(x)$ are real orthonormal eigenfunctions for the ground and first excited states of the oscillator, respectively.

a) [5 pts] Find the angular frequency, ω, at which the probability density $|\psi(x,t)|^2$ oscillates.

b) [5 pts] Find the expectation value for the energy.

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3. [10 pts] The quantum mechanical position operators, x, y, z and momentum operators, p_x, p_y, p_z, acting on a wavefunction ψ result in a specific value for the canonical commutation relations: $\left[x, p_x\right], \left[y, p_y\right]$, and $\left[z, p_z\right]$.

a) [5 pts] What is the value for each of these commutation relations?

b) [5 pts] Find a value for $\left[x^2, p_x\right]$

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4. [10 pts] The temperature of the cosmic microwave background is ~3K.

a) [2 pts] What is the typical energy of a cosmic ray photon?

Parts b), c), and d) of this problem involve the collision of a very high-energy proton ($m_p \sim 940$ MeV/c^2) with a cosmic ray photon to produce a pion ($m_\pi \sim 140$ MeV/c^2) and a scattered proton.

b) [1 pt] At the minimum energy of the proton for this reaction to occur (*i.e.*, at threshold) what is the direction of the proton with respect to the direction of the photon?

c) [2 pts] At threshold, in the proton-photon center-of-momentum frame of the collision, what is the total energy of the system?

d) [5 pts] In the frame of the universe where the photons have the typical energy (part a), what is the minimum energy of the proton for the reaction to occur?

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5. [10 pts] A thermoelectric cell is powered by a radioactive substance of half-life 100 days emitting β-particles of average energy, 5×10^{-14} joules . Assuming the cell to have the efficiency, 20%, calculate the amount (in gram-molecules) of radioactive substance required to generate 10 W of electricity.

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6. [10 pts] **Three** electrons are placed in a two-level system, with energies, $E_0 = 0$ and $E_1 = \varepsilon$. The Pauli exclusion principle for electrons of the same spin applies in each level. Assume the system has reached equilibrium at the temperature *T*. What is the average energy of the system?