SECRET STUDENT NUMBER: 54

DO NOT WRITE YOUR NAME OR STUDENT NUMBER ON ANY SHEET!

FUN FACTS TO KNOW AND TELL

$$\int_0^\infty dx\,\frac{x^{n-1}}{e^x-1} = \Gamma(n)\zeta(n) \qquad \int_0^\infty dx\,\frac{x^{n-1}}{e^x+1} = \Gamma(n)\zeta(n)\left[1-(1\ 2)^{n-1}\right]$$

$$\zeta(n) \equiv \sum_{m=1}^{\infty} m^{-n} \qquad \Gamma(n) \equiv (n-1)!$$

$$\zeta(3\ 2) = 2\ 612375 \qquad \zeta(2) = \frac{\pi^2}{6} \qquad \zeta(3) = 1\ 20205 \qquad \zeta(4) = \frac{\pi^4}{90}$$

$$\int_{-\infty}^{\infty} dx\, e^{-x^2\ 2} = \sqrt{2\pi} \qquad \int_0^\infty dx\, x^n e^{-x} = n!$$

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LONG ANSWER SECTION

1. (10 pts) Consider two single-particle energy levels, 0 and ϵ. Spin-1 bosons ($m = -1\ 0\ 1$) are allowed to populate the levels and equilibrate with a heat and particle bath de ned by a temperature T and chemical potential $\mu < 0$. The bosons are indistinguishable aside from their spin. What is the average number of bosons in each level?

2. (10 pts) Assume that the free energy in a two-dimensional system obeys the following form,

$$F = \int d^2r \left\{ \frac{A}{2}\phi^2 + \frac{C}{2}\phi^6 \right\}$$

Assuming that near T_c, $A \sim at$, nd the critical exponent in mean eld theory β where,

$$\langle \phi \rangle \sim t^\beta$$

below T_c.

3. N ink molecules are placed in a liquid at a time $t = 0$ and diffuse according to a diffusion constant D, i.e., the density of molecules satis es the diffusion equation,

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2}$$

For example, if the N molecules are initially positioned at $x = 0$ in a translationally-invariant medium, the density evolves as,

$$\rho(x\ \ t) = \frac{N}{\sqrt{4\pi D t}} e^{-x^2\ \ 4Dt}$$

(a) (10 pts) Now, add an absorptive boundary at $x = 0$, and place the drop at a small distance a from the boundary. By *small* we will only consider times such that $2Dt >> a^2$. Solve for the density $\rho(x\ \ t)$. You should include only the lowest order in a.

(b) (5 pts) What fraction of molecules survive to time t? Again assume $2Dt >> a^2$.

4. Suppose the average energy E and the average number of particles N in a one-dimensional system of extent L are given as a function of T, L and $\alpha \equiv -\mu\ T$. Further assume that L is much larger than any microscopic scale or correlation length of the system.

   (a) (10 pts) Derive an expression for the speci c heat per unit length,

$$C \equiv \frac{1}{L}\frac{\partial E}{\partial T}\bigg|_N$$

   in terms of $T\ L\ E\ N\ \partial_T E_{\ \alpha}\ \partial_\alpha E_{\ T}$ and $\partial_\alpha N_{\ T}$.

   (b) (10 pts) Assume the correlations in the system are sufficiently local they can be expressed in terms of delta functions,

$$\begin{aligned}
\langle \Delta\rho(0)\Delta\rho(x)\rangle_{\alpha\,T} &= A\delta(x) \\
\langle \Delta\epsilon(0)\Delta\epsilon(x)\rangle_{\alpha\,T} &= B\delta(x) \\
\langle \Delta\epsilon(0)\Delta\rho(x)\rangle_{\alpha\,T} &= D\delta(x)
\end{aligned}$$

   where ϵ and ρ are the energy density and number density respectively. Express C in terms of T, α, A, B and D.

SHORT ANSWER SECTION

5. (1 pt each) Graph several isotherms on a P vs. V graph illustrating the characteristics of a liquid gas phase transition. The graph should include:

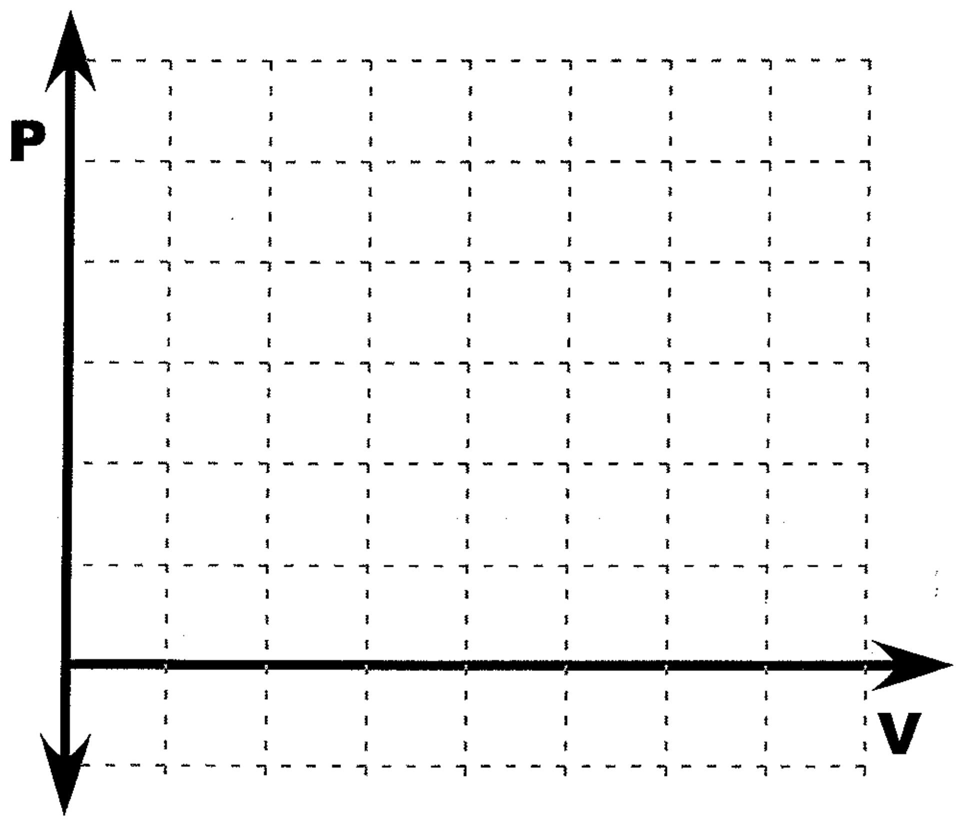
   (a) An isotherm with $T > T_c$.

   (b) An isotherm with $T = T_c$.

   (c) An isotherm with $T < T_c$.

   (d) Label the critical point.

   (e) For the isotherm with $T < T_c$, label the coexistence points.



6. (2 pts each) Consider a one-dimensional Ising model. Label each of the following statements as true or false.

   (a) In the exact solution there is no phase transition. __________

   (b) In the mean- eld solution there is no phase transition. __________

   (c) In the mean- eld solution, the critical exponents are the same for the one-dimensional and two-dimensional solutions. __________

9. (3 pts) One might expect a Goldstone boson from a phase transition with: (circle one)

   spontaneous breaking of a continuous symmetry

   spontaneous breaking of a discrete symmetry

   explicit breaking of a continuous symmetry

   explicit breaking of a discrete symmetry

10. (2 pts) For a system of massless bosons, $E = cp$, what dimensionality, D, is required for Bose condensation?

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